Problem IV
Smoothing weight

Assume that the weight production function is:
\[ w_{t+1} - w_t = \alpha_1 [-w_t + \theta c_t] \]
where \( c_t \) is consumption, that the asset equation is
\[ A_{t+1} - A_t = r_t A_t + y_t - p_t c_t \]
where \( y_t \) is income and \( p_t \) is price, that the utility function exhibits unit absolute risk aversion with respect to the index \( w_t \) of weight:
\[ U(w_t) = -\exp(-w_t) \]
and that the discount factor is \( \beta \). Assume further that prices alternate between \( p=2 \) and \( p=4 \), that \( \theta=1 \), that the interest rate \( r=0.1 \) and that the discount factor \( \beta=1/1.1 \).

1. Find \( w_{t+1} - w_t \) for \( \alpha_1 = 0.6 \) and also for \( \alpha_1 = 0.8 \).

2. Find \( c_{t+1} - c_t \) for \( \alpha_1 = 0.6 \) and also for \( \alpha_1 = 0.8 \).

3. Assuming that \( y=10 \) in each period, solve for \( w_t \) and \( c_t \) in each period. (Hint: You may assume without proof that \( A_{t+2} = A_t \), thus discounted consumption over two consecutive periods must equal discounted income over the same interval. Also, you will need a calculator--the answers are not in general whole numbers)

4. Why does an increase in the rate of depreciation result in a decrease in the volatility of weight?