

Problem IV
Smoothing weight

Assume that the weight production function is:

$$w_{t+1} - w_t = \alpha_1[-w_t + \theta c_t]$$

where c_t is consumption, that the asset equation is

$$A_{t+1} - A_t = r_t A_t + y_t - p_t c_t$$

where y_t is income and p_t is price, that the utility function exhibits unit absolute risk aversion with respect to the index w_t of weight:

$$U(w_t) = -\exp(-w_t)$$

and that the discount factor is β . Assume further that prices alternate between $p=2$ and $p=4$, that $\theta=1$, that the interest rate $r=0.1$ and that the discount factor $\beta=1/1.1$.

1. Find $w_{t+1} - w_t$ for $\alpha_1 = 0.6$ and also for $\alpha_1 = 0.8$.
2. Find $c_{t+1} - c_t$ for $\alpha_1 = 0.6$ and also for $\alpha_1 = 0.8$.
3. Assuming that $y=10$ in each period, solve for w_t and c_t in each period. (Hint: You may assume without proof that $A_{t+2}=A_t$, thus discounted consumption over two consecutive periods must equal discounted income over the same interval. Also, you will need a calculator--the answers are not in general whole numbers)
4. Why does an increase in the rate of depreciation result in a decrease in the volatility of weight?